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ABSTRACT

These materials were developed in the course of research on "Cognitive Growth through Peer Collaboration." Materials were developed and adapted for teaching spatial reasoning and logical reasoning to fourth grade students. The document contains: (1) descriptions of the various tasks involved; (2) outlines of each of the activities which were developed to teach the various tasks; and (3) pretests and post-tests used with a sample of fourth grade students used to gain feedback on the materials during their developmental stages. (TW)

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Cognitive Growth through Peer Collaboration

TASKS AND MATERIALS

Mathematics, Spatial Reasoning, and Balance Tasks

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Hopfenbeck, J., Mitchell, R. W., and Staven, K.

Radcliffe College and Clark University
January 1987

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Introduction

The following materials were developed in the course of our research, "Cognitive Growth through Peer Collaboration," which was funded by the Spencer Foundation, for the period from June, 1983 through December, 1986. William Damon (Department of Psychology, Clark University) was teh principal investigator and Erin Phelps (Murray Research Center, Radcliffe College) was the project co-director.

The development of materials was a process of adaptation, invention, pilot-testing, evaluation, and revision. The "first draft" was based on tasks used by other researchers. Through pilot-testing it was determined whether or not the materials were at an appropriate level of difficulty, would sustain the children's interest, and could be done by the children without an adult in the room. This last criterion became important when we noticed that the children's interactions were very different when an adult was present. On the other hand, we felt that the children needed feedback about the correctness of their performance throughout the intervention sessions.

For the first year of this project, materials were developed for a mathematics and a spatial reasoning series, each of which had six sessions. Within the domains of mathematics and spatial reasoning, each session was increasingly more difficult and built on the earlier sessions. For the second year, six sessions with a balance beam task were used to promote logico-mathematical reasoning. Problems in each session grew more difficult and required the use of more sophisticated concepts to successfully answer the questions.

During the development of intervention materials and pre- and post-tests, a sample of 10 fourth-graders provided feedback on the adequacy of the materials. This was judged by how the children reacted to the tasks, how much they talked as they worked together on the intervention problems, how long it took them to complete problems, the logistics of taping the interactions, and whether the tasks were too hard or too easy. On the basis of this experience, we discarded some tasks and revised others, and shortened the pre- and post-tests.

The nature of the three groups of tasks is briefly described below.

Mathematics tasks

Two kinds of math problems were developed -- missing addend and multiplication arithmetic problems, and ratio and proportionality problems. These were chosen because they represent a range of skills from those that fourth graders have had some experience with to those that are not usually taught until a later grade. In order to achieve our joint purposes of providing feedback about correct and incorrect answers and leaving the children to work without an adult present, we programmed a Radio Shack portable computer to present problems, evaluate answers, and record performance.

Missing addends and multiplication tasks were used for two sessions. In the first task, the children were presented with a set of 75 missing addend and multiplication problems. By using this familiar type of problem in the first task, the children were introduced to the new recording situation and dyadic format without also having to face a relatively novel task. The questions began with simple equations such as 3 + ? = 10 and 2 x 2 = ? As more problems were answered, however, the size of the numbers involved increased. In Task 2, a set of 49 problems of the same type were presented in word format instead of as numbers.

These multiplication and addition questions were presented to the children on a small portable computer with scratch paper and pencils available. When the children agreed on an answer, they entered it into the computer. If they were correct the computer beeped and displayed the next problem. If they were incorrect, the computer buzzed but allowed them another opportunity to try and answer correctly. Five consecutive wrong answers resulted in the automatic appearance of the next problem. All answers, correct and incorrect, were stored in the computer's memory and printed out after the session was complete.

Proportions tasks were used for four sessions. Proportions tasks 1 and 2 centered on tasks using ratios. A basic ratio pattern was presented on the computer screen and the children were asked three to four questions using the initial ratio. For example: "For every 5 nuts there are 5 bolts. How many nuts are needed for a) 10 bolts?, b) 7 bolts?, and c) 19 bolts?" Each problem was accompanied by a picture in Session 3 or a chart in Session 4, which illustrated the initial ratio. The children had scratch paper and pencils available to them. All answers were entered into the computer and, if they answered incorrectly five consecutive times, the next problem was automatically displayed on the screen.

Proportional reasoning problems were presented to the Math group in Proportions tasks 3 and 4. Red and white chips were used to present the concepts of discrete proportionality in the fifth session. Plastic chips were placed in the center of the table, along with two clear plastic boxes labelled A and B. An initial ratio was defined in the computer screen and the children set up the chips accordingly. They were then asked several question using the ratio. For example: "Box A has 1 red chip and 1 white chip. Box B has 1 red chip. How many white chips does Box B need to make

the mixture of the red and white chips the same as Box A?" If the pair missed the problem five consecutive times, the next problem automatically appeared on the screen. There were 29 problems available to be solved.

In Proportions task 4, reasoning with continuous quantities was presented through mixtures of water and juice (represented by colored water) in graduated cylinders. Problems were presented to the children on the small portable computer. The children were instructed to pour a specified mixture of juice and water into one cylinder. They were then asked to pour a specific amount of juice into the other cylinder and calculate how much water they would need to add to make the mixture taste the same as the mixture in the first cylinder. For example: "In beaker A put one cup juice and one cup water. Now in beaker B put one cup orange juice. How many cups of water does beaker B need to make the juice in beaker B taste like the juice in beaker A?" There were 15 problems available to be solved by the children.

Spatial reasoning tasks: The spatial reasoning tasks were chosen in order to provide a contrasting situation to that presented by the math tasks. They represent a domain of knowledge that has been extensively studied by psychologists interested in the development of reasoning, but are not part of the standard elementary school curriculum. In addition, as in other domains, important changes are beginning to occur in children's thinking when they are about 10 years old.

For the first spatial perspectives task, evaluation of performance was done with the computer, as in the math tasks. For the remaining five, evaluation could not be done in a piecemeal fashion because there were not discrete items. In these cases, the experimenter would go in near the end of a session and ask questions about the children's product, pointing out problems and inconsistencies. Then the children would have a few more minutes to respond to the critique and correct whatever they wished to.

Model building tasks were given in two of the spatial reasoning sessions. In Model building tasks 1 and 2, the children constructed models using LEGO bricks (this task was developed and pilot-tested by the project staff). The children were given drawings of multicolored objects which they then had to build using different colored building blocks. In the first task, the children were presented with a picture of a yellow and red wall on a blue base. The picture was not drawn to the same scale as the bricks the children used, so that they were unable to place the bricks directly on the picture and duplicate it. After being handed the blue Lego to serve as the base, the children were asked to build the rest of the wall on the base so that it "looked just like the drawing." They were asked to duplicate the wall color pattern.

In task 2, children were required to construct a house from two drawings. They were given a front and a rear view of a house, not drawn to scale, and asked to build the house using the Lego placed in front of them.

Spatial perspectives tasks were presented for four sessions. For the first task, the children were presented with two models, a village scene and a more abstract urban scene. Around each scene were placed eight colored figures ("dolls"). From their chairs, the children had to picture what the view was from each doll's perspective. They then chose the correct perspective from fifteen pictures of each model, eight of which were correct and the remainder, false views. Each picture was marked on its back with a letter which the children entered into the portable computer. If they answered incorrectly three consecutive times, the next question (e.g. "What does the red doll see?") automatically appeared on the screen.

In Task 2 the children built a copy of a model from a different perspective. The children stood between two tables, one placed at a 90 degree angle to the other and each with a base board for a model on it. Each problem had two parts. First the children used a photograph and scale size chart to assemble a scene using wooden blocks and trees on one base board. For the second part they copied the model onto the second base board which was turned a different way.

Each board was painted with a large blue lake and green grass surrounding the water. The children were given the following instructions to guide their copy construction. "Look at the model you've set up. Pretend that you are swimming in the lake. When you get out, you see these houses. Now, I want you two to set up houses and trees on the other board so that if you got out of this lake the same way, you would see the houses in the same way." The experimenter stayed in the room while the children attempted a sample problem to insure that they understood the task correctly. After each task problem the experimenter entered the room briefly to rotate the base board and trace around the children's solution.

The complexity of the task was varied by changing the orientation of the copy base and by changing the number of buildings and trees. In simple items, the orientation of the copy base was the same as the model base and the number of buildings and trees was small. In more complicated items, the base was rotated and the number of buildings and trees was increased.

In spatial perspectives tasks 3 and 4, the children collaborated on drawing maps of two models. Different models (suburban and rural) were used for each session. The model was placed in front of the students and they were handed one 12" x 12" sheet of paper and two standard lead pencils with erasers. The students were allowed to walk up and examine the model, but drew the map from their original seats. They were instructed to draw a map, not just draw a picture of the scene, but were allowed to interpret this command without further instruction.

Near the end of the session, the experimenter entered the room to give the children feedback on what they had drawn and to make suggestions for additions. Then the children were give a few more minutes to work on their map.

Balance task: In this task, children must predict which arm of a balance will go down when various configurations of weights and distances are set up. The balance scale apparatus had five pegs on each side of the fulcrum upon which metal washers were placed. Depending upon the placement and number of weights used, the arm could tip left, right or remain balanced. Blocks of wood or paper cups were placed under each side to keep it stationary while the children set up the problem as indicated in drawings of each problem in a binder. Once the weights were arranged the children had to predict which side, if any, would go down when the blocks or cups were removed.

The balance task was chosen for several reasons. The task used both the concept of proportionality and a definite mathematical formula for generating correct solutions. Because of this, the Math group's balance scores would be of particular interest in relation to the Spatial and Control groups. In addition, there is a body of empirical work using the scale, so that its efficacy as a learning/theoretical tool is already established. Scale problems have a definite hierarchy in terms of difficulty and rules needed to successfully predict the outcome. This allowed for the introduction of progressively harder problems.

The simplest types of problems were basic balance, weight and distance problems. Equal weights on matching opposing pegs (balance), unequal weights on matching opposing pegs (more weight), and equal weights on nonmatching opposing pegs (greater distance) were all fairly easy for most of the children. When the weights and pegs were varied beyond these simple mixtures, however, performances were much more inconsistent. These more difficult conflict-weight, conflict-distance, and conflict-balance problems were mixed in with the easier problems in ever increasing numbers, until the final sessions primarily consisted of more difficult problems.

The children were given a booklet of 10 problems. For each one, they set up the balance scale with the arms blocked from moving. Then they were asked to predict which arm, if any, would go down and to write down their answer when they agreed. At that point, they removed the blocks or cup from under the scale arms in order to see what in fact would happen. If they had answered correctly, they circled their answer on the paper. In this way, they were provided feedback about their performance directly from the materials.

After the children had completed 10 problems, the booklet told them to stop. At this point an investigator entered to room and told them to try again the problems that they had missed. In addition, they were asked to experiment with their own combinations in order to try to find out how the balance worked. After 5 minutes, the investigator returned and asked them to explain how the balance worked. After receiving an explanation or a clear statement of "We don't know," the experimenter gave them the booklet with 10 additional problems and asked them to do as many as they could in the remaining time. The total session lasted until they had completed the remaining problems or until twenty minutes had elapsed.

Children wrote their answers on a piece of paper. These answers were scored after every session. Any problems the children answered incorrectly were repeated in following sessions until they were answered correctly. Additional problems were added from a list of progressively harder problems, so that each session presented 20 items.

Pre-tests and post-tests: On the basis of the task materials, pre- and post-tests were devised. They included some items that were easier than the intervention tasks to make sure that the children had the basic skills, some items that were identical to the intervention material, and other items that were different from but of the same difficulty as the intervention. The math tests consisted of 68 problems which the children were given 30 minutes to complete. The spatial reasoning test consisted of 4 stations around which the children rotated. Each station contained a spatial display or a picture of a display and the child had to perform a task that was explained by an adult at each station. The total time allowed for the spatial reasoning test was again 30 minutes. For the balance test, children received worksheets with 30 items. The items pictured a scale with weights placed on it and the children had to indicate whether the scale was balanced or which side would go down. In addition, the experimenter stood before the children with a real scale and weights and set up each item for the children.

Mathematics Tasks

Missing Addends and Multiplication Problems -- Tasks 1 and 2

Sources:

Gold (1978); Case (1980, 1982); Kilian, et. al. (1980); Parkman (1972); Vergnaud (1983); various fourth-grade math texts

Materials:

Computer
Scratch paper
Pencil

Items to be used:

75 missing addend and multiplication problems, in numerical format, for session 1.
49 missing addend and multiplication problems, in word format, for session 2.

Instructions:

Seat subjects at table with computer before them. Provide scratch paper and pencils within reach.

"Today you are going to work together on some math problems. They will appear on this computer and you will answer them by typing the answer in to the computer. When we start the problems, the computer will present some instructions to you. Please read aloud all instructions given by the computer and all problems. Then you do the first two problems for practice. You can use the paper and pencils to help figure out the answers. We will stay here until you have done the first two problems and then we'll leave you to work alone."

"Okay, when you do these problems, there are a couple of important things I want you to do. First, please work <u>together</u>. Second, wait until you both agree that a problem is finished and correct before you go to the next problem. It doesn't matter how <u>many</u> problems you do, but how well you do them. The computer will keep a record for us of your answers. The last thing is, when you talk to each other, please speak up so that your conversations get recorded."

"Do you have any questions? (Answer questions.) Then you can start."

Scoring:

Number correct.

Missing Addends and Multiplication Task 1 -- Problems -- Numerical Format

3 + ? = 10	? + 5 = 17	18 = ? + 9	12 = 4 + ?
? + 6 = 17	30 = ? + 22	45 = 6 + ?	8 + ? = 79
2 X 2 = ?	1 X 5 = ?	4 X 6 = ?	3 X 9 = ?
7 X 2 = ?	8 X 5 = ?	9 X 9 = ?	6 X 7 = ?
8 X 6 = ?	? + 89 = 95	56 = ? + 47	28 = 15 + ?
39 + ? = 52	? + 40 = 66	75 = ? + 36	88 = 46 + ?
13 X 2 = ?	11 X 7 = ?	62 X 3 = ?	74 X 2 = ?
81 X 7 = ?	90 X 6 = ?	15 X 3 = ?	25 X 8 = ?
86 X 4 = ?	78 X 9 = ?	57 + ? = 103	? + 87 = 112
130 = ? + 71	104 = 76 + ?	59 + ? = 108	? + 87 = 243
159 = ? + 49	223 X 3 = ?	312 X 2 = ?	301 X 6 = ?
611 X 5 = ?	415 X 4 = ?	125 X 8 = ?	203 X 9 = ?
765 X 4 = ?	804 X 9 = ?	645 X 8 = ?	256 = 194 + ?
821 + ? = 882	? + 611 = 649	731 = ? + 149	362 = 150 + ?
387 + ? = 537	? + 250 = 625	375 = ? + 122	724 = 327 + ?
505 + ? = 839	? + 399 = 672	831 = ? + 287	963 = 197 + ?
12 X 11 = ?	31 X 32 = ?	41 X 73 = ?	35 X 30 = ?
23 X 90 = ?	24 X 85 = ?	73 X 50 = ?	87 X 12 = ?
78 X 64 = ?	86 x 68 = ?	672 + ? = 1071	

Missing Addends and Multiplication Task 2 -- Problems -- Word Format

1. How many chairs are there if Tom needs to put 2 more at the table so each of his 8 guests will have a seat? (6)

2. How many fish does Laurie have if Charles has 7 fish which is 3 more than Laurie has? (4)

3. If a relay team has 4 runners, and there are 7 relay teams, how many runners are there altogether? (28)

4. Jessica jumps rope 5 minutes every day. How many minutes does she jump in 7 days? (35)

5. Three teams won trophies for swimming. If each team won 2 trophies, how many trophies were won in all? (6)

6. Three pencils were given to each of 9 students. How many pencils were given altogether? (27)

7. Together, Susan and Ryan have 83 comic books. If Ryan has 37 comic books, how many does Susan have? (46)

8. The math homework had 4 rows of 6 problems each. Then there were 3 story problems. How many problems were there altogether? (27)

9. In the school paper drive, Mrs. Brown's class collected 127 pounds of newspaper. Mr. Smith's class got 93 lbs. How much more does Mr. Smith need to have as much as Mrs. Brown? (27)

10. Frank lost two coins through a hole in his pocket. First he lost 5 cents. When he got to school, he was short 30 cents. If he started out with 65 cents, how much did he lose the second time? (25)

11. Terry received 2 sets of stickers for her birthday which added 34 to her collection. The first set contained 23 stickers. If she now has 60 stickers, how many were in the second set? (11)

12. Beth and Andy sold 82 boxes of candy last weekend. They sold 47 boxes on Saturday and 35 on Sunday. If Andy sold 23 boxes, how many did Beth sell in all? (59)

13. Jon lost 33 lbs. during the first 2 months of a diet. In the 2nd month he lost a total of 17 lbs. At the start of his diet, Jon was 69 lbs. overweight; now he is only 36 lbs. overweight. How many lbs. did he lose the first month? (16)

14. Six soccer teams were invited to play in a tournament. If there are 11 players on a team, how many t-shirts should be ordered for the players? (66)

15. George rakes leaves to earn money for a new bike. If he rakes enough leaves to fill 5 baskets in 1 hour, how many baskets does he fill in 30 hours? (150)

16. Mrs. Taber can do 21 haircuts a day. She works 5 days a week. How many haircuts does she give in a week? (105)

17. Notebooks were on sale for 48 cents each. How much does Terry spend for 4 notebooks? ($1.92)

18. If pencils are 15 cents each, how much will 7 pencils cost? ($1.05)

19. Jay walks 1 lap in 9 minutes. How many minutes does it take him to walk 24 laps? (216)

20. In their last game, the United States basketball team scored 23 points in each quarter. How many points were scored in all 4 quarters? (92)

21. Jamie sold 12 bags of cookies. If each bag has 3 chocolate chip and 4 peanut butter cookies, how many cookies did he sell in all? (84)

22. Jeff's Market received 7 boxes of apples. How many apples did they receive if there were 25 apples in each box? (175)

23. There are 46 rows in a movie theater with 9 seats in each row. If all the seats are filled, how many tickets were sold? (414)

24. How many miles has a leopard run before running 41 more miles to reach an eagle that had flown 115 miles? (74)

25. Fran and Tom inspected 125 eggs and found 35 were cracked. Tom found and threw away 16 cracked eggs. After the inspection, there were 90 eggs left. How many did Fran throw out? (19)

26. Doug asked Lisa and Craig to help him deliver 225 papers. Doug left 65 papers with Craig and kept 85 papers for himself. If Doug ended up delivering all but 140 of the papers, how many did he leave with Lisa? (75)

27. Michelle has 3 stamp albums, each with 147 stamps. She received another 15 stamps for her birthday. How many stamps does she have in her collection now? (192)

28. If eggs sell for $1.29 a dozen, how much will 4 dozen cost? ($5.16)

29. If a train ticket costs $4.92, how much does Mrs. Smith pay for 3 tickets? ($14.76)

30. There are 144 eggs in a crate. If there are 8 crates, how many eggs are there in all? (1152)

31. A jet averages 878 kilometers an hour. How far does it travel in 4 hours? (3512)

32. Miss Owens pays a helper $3.95 an hour to sweep her shop. How much does she pay for 5 hours' work? ($19.75)

33. A meteor traveled at 986 miles per hour. If it travelled for 9 hours before burning up, how many miles did it cover? (8874)

34. Paula rode 181 miles on 2 different buses. On the first, she went 146 miles. At the start of the trip, Paula was 195 miles from home; now she is 14 miles away. How far did she go on the second bus? (35)

35. The Chase family bought 8 storm windows and 9 rolls of insulation. If each window cost $39 and each roll of insulation cost $8.50, how much did they spend? ($388.50)

36. At the end of Unit 4, Ann's math team had 164 more points than at the end of Unit 3. At the end of Unit 4 they had 379 points. What was their total score at the end of Unit 3? (215)

37. At Jan's Bakery, Kevin and Fred bake 720 pies a month. 415 of the pies are apple and 305 are cherry. If Kevin makes 371 pies, how many does Fred make in all? (349)

38. Jane bought 675 feet of blue and red ribbon. She used 350 feet to make party decorations. If 249 feet of the ribbon she bought were blue, how many feet were red? (426)

39. There are 36 robots, each with 24 different switches. How many switches are there altogether? (864)

40. An artist drew a set of 18 pictures. How many pictures did he draw if he made a total of 16 sets? (288)

41. This year Joe's Fish Company caught 347 more lbs. of fish than last year. This year they caught 2583 lbs. What was the total number of lbs. caught last year? (2236)

42. In 1983, Turner Farm grew 1341 bushels of corn. 400 bushels were used as animal feed; the rest was made into corn meal. In 1984, 2716 bushels were grown. What is the total number of bushels gained in 1984? (1375)

43. Adams Department Store ordered 30 boxes of wool scarves. If there are 144 scarves in each box, how many scarves did the store receive? (4320)

44. Skyline Drive Center contains 12 office buildings. There are 225 people working in each building. How many people work at Skyline Center? (2700)

45. A movie theater sold 8527 tickets last week. 6738 tickets were sold on Saturday. That is 4949 more than were sold on Sunday. Altogether, how many tickets were sold on Sunday? (1789)

46. A flower shop ordered 23 boxes of carnations and 40 boxes of roses. If each box of carnations has 5 dozen carnationa and each box of roses has 7 dozen roses, how many dozen flowers did the shop order? (395)

47. This year, 7429 babies were born in Danville. Last year, there were 625 more boys than girls. If 3015 of the babies born this year were boys, in all, how many were girls? (4414)

48. Walter delivered 8 maple trees to Mrs. Abels. Each maple tree costs $32.50. How much money did Mrs. Abels pay Walter? ($260.00)

49. Gail used 3 packages of blue beads and 4 packages of green beads. Each package had 12 beads. How many beads did Gail use? (84)

Proportions Tasks 1 and 2 -- Ratios

Sources:

Behr, et. al. (1983); Kieren & Nelson (1978); Kieren & Southwell (1979); various fourth-grade and fifth-grade math texts

Materials:

Computer
Pictures for set 1
Charts for set 2
Scratch paper
Pencil

Items to be used:

35 items for set 1, attached.
40 items for set 2, attached.

Instructions:

Seat subjects at table with computer before them. Provide scratch paper and pencils within reach.

"Today you are going to work together on some math problems. They will appear on this computer and you will answer them by typing the answer in to the computer. When we start the problems, the computer will present some instructions to you. Please read aloud all instructions given by the computer and all problems. Then you can try the first problem for practice. Each problem has a picture/chart to go with it. You can use the paper and pencils to help figure out the answers. We will stay here until you have done the first problem and then we'll leave you to work alone."

"Okay, when you do these problems, there are a couple of important things I want you to do. First, please work <u>together</u>. Second, wait until you both agree that a problem is finished and correct before you go to the next problem. It doesn't matter how <u>many</u> problems you do, but how well you do them. The computer will keep a record for us of your answers. The last thing is, when you talk to each other, please speak up so that your conversations get recorded."

"Do you have any questions? (Answer questions.) Then you can start."

Scoring:

Number correct.

Proportions Task 1 -- Problems -- With Pictures

1. For every 5 nuts, there are 5 bolts. How many nuts are needed for --
   a) 10 bolts? (10) b) 7 bolts? (7)
   c) 19 bolts? (19)

2. An ice cream store gives away 2 pencils with every ice cream cone it sells. How many pencils are given with --

   a) 2 cones? (4) b) 3 cones? (6)
   c) 5 cones? (10)

3. Every time Tom eats 3 cookies, he drinks a glass of milk. How many glasses of milk does he drink with --

   a) 6 cookies? (2) b) 9 cookies? (3)
   c) 15 cookies? (5)

4. A locksmith uses 4 screws for each lock he installs. How many screws does he need for --

   a) 2 locks? (8) b) 4 locks? (16)
   c) 7 locks? (28)

5. Every shirt made by the Harris Company has 5 buttons. How many buttons are needed for --

   a) 3 shirts? (15) b) 5 shirts? (25)
   c) 10 shirts? (50)

6. Kevin is making signs for a car wash. He uses 2 cans of spray paint to make 3 signs. How many signs can he make with --

   a) 4 cans? (6) b) 8 cans? (12)
   c) 12 cans? (18) d) 14 cans? (21)

7. To make dark green paint, the paint store mixes 2 cans of yellow paint with 6 cans of blue paint. How many cans of blue paint should be mixed with --

   a) 4 cans of yellow paint? (12)
   b) 6 cans of yellow paint? (18)
   c) 10 cans of yellow paint? (30)
   d) 1 can of yellow paint? (3)

8. At the Fireside Inn, Mark puts 2 red flowers and 1 yellow flower in a vase on each table. How many yellow flowers does he need if he has --

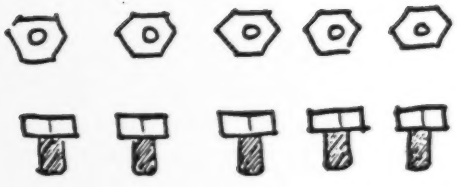
   a) 4 red flowers? (2)
   b) 6 red flowers? (3)
   c) 10 red flowers? (5)
   d) 12 red flowers? (6)

9. Sharon is making some necklaces. Each necklace has 4 blue beads and 6 purple beads. How many purple beads does she use if she uses --

   a) 9 blue beads? (15)
   b) 12 blue beads? (20)
   c) 21 blue beads? (35)
   d) 24 blue beads? (40)

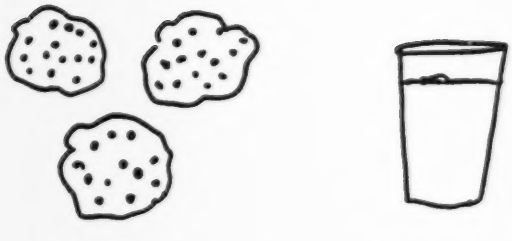
Pictures for Proportions Task 1

1.



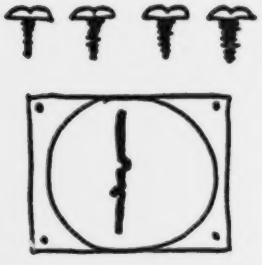
2.

3.



Pictures for Proportions Task 1

4.



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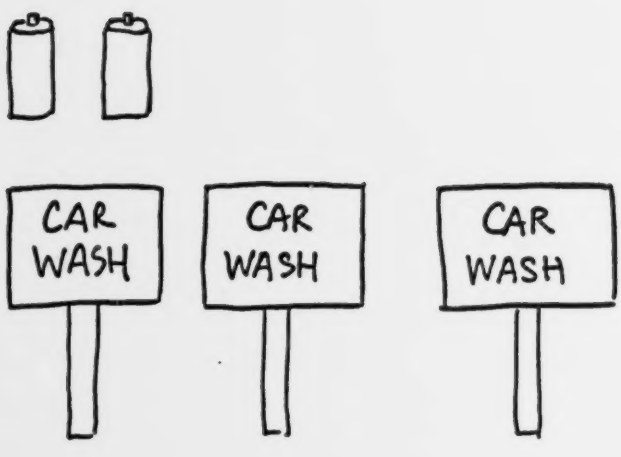
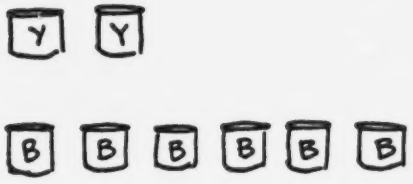


Pictures for Proportions Task 1

7.



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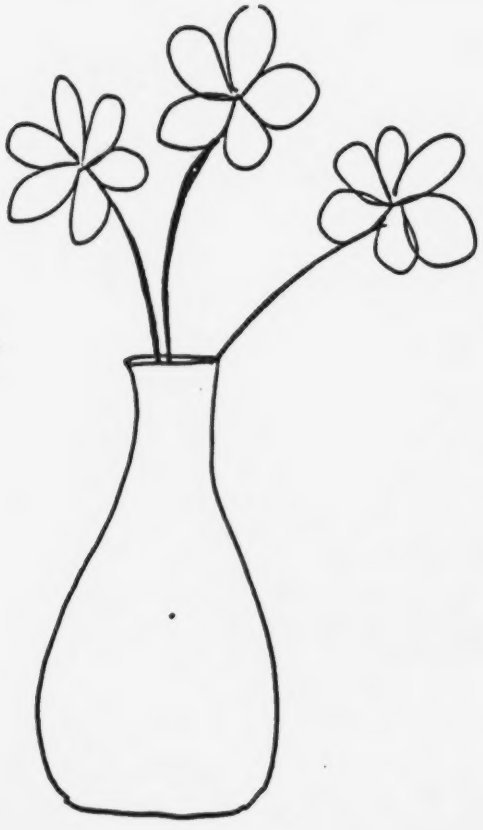

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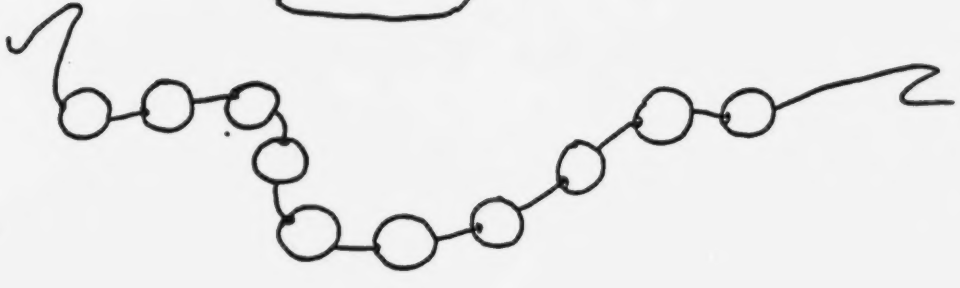


Proportions Task 2 -- Problems -- With Charts

1. Each week, Kim saves $2 of her allowance. How much money does she save in --

   a) 2 weeks? (4) b) 3 weeks? (6)
   c) 4 weeks? (8) d) 8 weeks?(16)

2. Terry's scout troop is planning a hiking trip. If they hike 3 miles in one hour, how long will it take them to hike --

   a) 6 miles? (2) b) 9 miles? (3)
   c) 12 miles? (4) d) 18 miles? (6)

3. If 1 dime equals 2 nickels, how many dimes equal --

   a) 4 nickels? (2) b) 6 nickels? (3)
   c) 8 nickels? (4) d) 12 nickels? (6)

4. Valerie's class is helping to move some boxes in the school library. If each child moves 3 boxes, how many boxes will be moved by --

   a) 2 children? (6) b) 3 children? (9)
   c) 5 children? (15) d) 10 children? (30)

5. John is emptying bags of candy into a dish. Each bag has 2 green candies and 4 yellow candies. How many yellow candies are in the dish if there are --

   a) 4 green candies? (8) b) 6 green candies? (12)
   c) 8 green candies? (16) d) 9 green candies? (18)

6. Jane makes salad dressing with 6 tablespoons of oil and 2 tablespoons of vinegar. How many tablespoons of vinegar does she need with --

   a) 12 tablespoons oil? (4) b) 18 tablespoons oil? (6)
   c) 21 tablespoons oil? (7) d) 24 tablespoons oil? (8)

7. Tim is making some cupcakes. He uses 4 cups of flour and 2 cups of sugar. How much sugar does he need to use with --

   a) 8 cups of flour? (4) b) 16 cups of flour? (8)
   c) 18 cups of flour? (9) d) 20 cups of flour? (10)

8. If 2 pencils cost 5 cents, how much money do you need for --

   a) 4 pencils? (10) b) 6 pencils? (15)
   c) 8 pencils? (20) d) 12 pencils? (30)

9. The Nut House puts 3 lbs. of almonds and 5 lbs. of peanuts in its nut mixture. How many lbs. of peanuts will there be if there are --

   a) 6 lbs. of almonds? (10) b) 9 lbs. of almonds? (15)
   c) 12 lbs. of almonds? (20) d) 18 lbs. of almonds? (30)

10. Tina is mixing lemonade and limeade together to make punch for the school picnic. She uses 2 cups limeade and 3 cups lemonade. How much lemonade should she use with --

   a) 4 cups limeade? (6) b) 6 cups limeade? (9)
   c) 10 cups limeade? (15) d) 14 cups limeade? (21)

Charts for Proportions Task 1

1.

WEEKS	1	2	3	4	8
AMOUNT SAVED	2				

2.

MILES	3	6	9	12	18
HOURS	1				

3.

NICKELS	2	4	6	8	12
DIMES	1				

4.

CHILDREN	1	2	3	5	10
BOXES	3				

5.

GREEN CANDIES	2	4	6	8	9
YELLOW CANDIES	4				

Charts for Proportions Task 1

6.

OIL	6	12	18	21	24
VINEGAR	2				

7.

FLOUR	4	8	16	18	20
SUGAR	2				

8.

PENCILS	2	4	6	8	12
COST	5c				

9.

ALMONDS	3	6	9	12	18
PEANUTS	5				

10.

LIMEADE	2	4	6	10	14
LEMONADE	3				

Proportions Task 3 -- Discontinuous Materials, Chips

Sources:

Gold (1978); Karplus, Pulos & Stage (1983)

Materials:

2 clear plastic boxes, labeled A and B
50 red poker chips
50 white poker chips
Computer
Scratch paper
Pencil

Items to be used:

29 items, attached.

Instructions:

Subjects are seated at table with computer between them. Pile poker chips in center of table within easy reach of both children.

"Today you are going to work together on some math problems with the chips. They will appear on this computer and you will answer them by typing the answer in to the computer. When we start the problems, the computer will present some instructions to you. Please read aloud all instructions given by the computer and all problems. Then you do the first three problems for practice. You can use the paper and pencils to help figure out the answers. We will stay here until you have done the first three problems and then we'll leave you to work alone."

If they really do not understand (i.e., only as a last resort), explain the correct solution using the Between strategy. For problems 1 and 2, tell them, "There is the same number of white chips as red chips in Box A. So in Box B there should be the same number of white chips as red chips." For problem 3 say, "The number of white chips in Box A is twice as many as the number of red chips in Box B."

"Okay, when you do these problems, there are a couple of important things I want you to do. First, please work together. Second, wait until you both agree that a problem is finished and correct before you go to the next problem. It doesn't matter how many problems you do, but how well you do them. The computer will keep a record for us of your answers. The last thing is, when you talk to each other, please speak up so that your conversations get recorded."

"Do you have any questions? (Answer questions.) Then you can start."

Scoring:

Number correct.

We may also code for presence/absence of a proportional reasoning strategy. This is done by considering explanations of how they arrived at their solution. Gold (1978) identified the following solution strategies:

1. Use of multiplication or division, e.g., "There's twice (or half) as many white ones so there should be twice (half) as many red ones."

2. Unit or "for every" strategy, e.g., "There's two white ones for every red one."

Proportions Task 3 -- Problems -- Chips

1. BOX A has 1 red chip and 1 white chip. BOX B has 1 red chip. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (1)

2. BOX A has 1 red chip and 1 white chip. BOX B has 2 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

3. BOX A has 2 red chips and 4 white chips. BOX B has 4 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (8)

4. BOX A has 3 red chips and 3 white chips. BOX B has 6 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (6)

5. BOX A has 5 red chips and 5 white chips. BOX B has 3 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (3)

6. BOX A has 1 red chip and 1 white chips. BOX B has 2 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (4)

7. BOX A has 2 red chips and 4 white chips. BOX B has 4 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (8)

8. BOX A has 3 red chips and 6 white chips. BOX B has 7 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (14)

9. BOX A has 5 red chips and 10 white chips. BOX B has 6 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (12)

10. BOX A has 1 red chip and 3 white chips. BOX B has 2 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (6)

11. BOX A has 2 red chips and 6 white chips. BOX B has 4 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (12)

12. BOX A has 3 red chips and 9 white chips. BOX B has 5 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (15)

13. BOX A has 1 red chip and 3 white chips. BOX B has 3 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (9)

14. BOX A has 2 red chips and 1 white chip. BOX B has 4 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

15. BOX A has 4 red chips and 2 white chips. BOX B has 8 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (4)

16. BOX A has 6 red chips and 3 white chips. BOX B has 14 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (7)

17. BOX A has 10 red chips and 5 white chips. BOX B has 12 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (6)

18. BOX A has 3 red chips and 1 white chip. BOX B has 6 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

19. BOX A has 6 red chips and 2 white chips. BOX B has 3 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (1)

20. BOX A has 9 red chips and 3 white chips. BOX B has 6 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

21. BOX A has 3 red chips and 1 white chip. BOX B has 9 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (3)

22. BOX A has 6 red chips and 3 white chips. BOX B has 2 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (1)

23. BOX A has 8 red chips and 4 white chips. BOX B has 4 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

24. BOX A has 6 red chips and 2 white chips. BOX B has 3 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (1)

25. BOX A has 12 red chips and 4 white chips. BOX B has 6 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

26. BOX A has 6 red chips and 2 white chips. BOX B has 12 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (4)

27. BOX A has 4 red chips and 2 white chips. BOX B has 2 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (1)

28. BOX A has 2 red chips and 4 white chips. BOX B has 1 red chip. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (2)

29. BOX A has 1 red chip and 3 white chips. BOX B has 2 red chips. How many white chips does BOX B need to make the mixture of red and white chips the same as BOX A? (6)

Proportions Task 4 -- Continuous Materials, Juice

Sources:

Gold (1978); Karplus, Pulos & Stage (1983)

Materials:

2 one-liter graduated cylinders, labeled A and B
Pitcher of plain water
Pitcher of colored water, representing juice
Large bucket
Computer
Scratch paper
Pencil

Items to be used:

15 items, attached.

Instructions:

Subjects are seated at table with computer, cylinders, and pitchers of colored and plain water in front of them. The scratch paper and pencils are off to one side. The bucket is somewhere on the floor (behind the subjects if possible), within easy reach of both subjects.

"Today you are going to work together on some math problems with the water and cylinders. They will appear on this computer and you will answer them by typing the answer in to the computer. When we start the problems, the computer will present some instructions to you. Please read aloud all instructions given by the computer and all problems. Then you do the first three problems for practice. You can use the paper and pencils to help figure out the answers. We will stay here until you have done the first three problems and then we'll leave you to work alone."

"You will be working on problems about making juice. It isn't really juice, but just water with food coloring in it. After you finish each problem, you should empty water from the cylinders into the bucket. Then you can start the next problem."

If they really do not understand (i.e., only as a last resort), explain the correct solution using the Between strategy. For problems 1 and 2, tell them, "there is the same amount of water as juice in cylinder A. So in cylinder B there should be the same amount of water as juice." For problem 3 say, "The amount of water in cylinder A is twice as much as the amount of juice. So the amount of water in cylinder B should be twice the amount of juice in cylinder B."

"Okay, when you do these problems, there are a couple of important things I want you to do. First, please work together. Second, wait until you both agree that a problem is finished and correct before you go to the next problem. It doesn't matter how many problems you do, but how well you do them. The computer will keep a record for us of your answers. The last thing is, when you talk to each other, please speak up so that your conversations get recorded."

"Do you have any questions? (Answer questions.) Then you can start."

Scoring:

Number correct.

We may also code for presence/absence of a proportional reasoning strategy. This is done by considering explanations of how they arrived at their solution. Gold (1978) identified the following solution strategies:

1. Use of multiplication or division, e.g., "There's twice (or half) as much water so there should be twice (half) as much juice."

2. Unit or "for every" strategy, e.g., "There's two waters for every juice."

Proportions Task 4 -- Problems -- Juice

1. Pitcher A has 1 cup orange juice and 1 cup water. Pitcher B has 1 cup orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (1)

2. Pitcher A has 2 cups orange juice and 2 cups water. Pitcher B has 1 cup orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (1)

3. Pitcher A has 1 cup orange juice and 2 cups water. Pitcher B has 2 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (4)

4. Pitcher A has 1 cup orange juice and 1 cup water. Pitcher B has 3 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (3)

5. Pitcher A has 2 cups orange juice and 2 cups water. Pitcher B has 1 cup orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (1)

6. Pitcher A has 3 cups orange juice and 3 cups water. Pitcher B has 5 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (5)

7. Pitcher A has 1 cup orange juice and 2 cups water. Pitcher B has 3 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (6)

8. Pitcher A has 2 cups orange juice and 1 cup water. Pitcher B has 4 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (2)

9. Pitcher A has 3 cups orange juice and 6 cups water. Pitcher B has 2 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (4)

10. Pitcher A has 1 cup orange juice and 3 cups water. Pitcher B has 2 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (6)

11. Pitcher A has 2 cups orange juice and 6 cups water. Pitcher B has 1 cup orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (3)

12. Pitcher A has 3 cups orange juice and 1 cup water. Pitcher B has 6 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (2)

13. Pitcher A has 2 cups orange juice and 4 cups water. Pitcher B has 3 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (6)

14. Pitcher A has 4 cups orange juice and 1 cup water. Pitcher B has 8 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (2)

15. Pitcher A has 6 cups orange juice and 2 cups water. Pitcher B has 3 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A? (1)

Spatial Reasoning Tasks

Model Building Tasks 1 and 2

Materials:

Drawings of model (view of wall for session 1, and 2 views of house for session 2)
Legos
Scoring template showing correct placement of Legos in wall

Items to be Used:

Copies of drawings.

Instructions:

WALL: "Today you are going to work together on building a wall out of Legos. Here is a drawing of the way the completed wall should look. Use the Legos you have in front of you to build it. This blue piece I am handing you is the blue base you see in the drawing. Build the rest of the wall on this blue piece so that it looks just like the drawing. The front of the wall must be flat and smooth but the back can have pieces sticking out of it (show example).

HOUSE: "Today you are going to work together on building a house out of Legos. Here are two drawings of the house you will be building. This is how it looks from the front (point to it), and this is how it looks from the back (point). Use the Legos in front of you to build a house that looks just like the one in the pictures.

BOTH: "When you build this, there are a couple of important things I want you to do. First, please work together on this problem until you both think you have finished it correctly. Second, when you talk to each other, please speak up so that your conversations get recorded. Do you have any questions? (Answer them.) Then you can start."

Scoring for House:

Five parts are each scored on a 0 - 4 scale. These scores are then summed for a final total score. The parts are: (1) ends with doors; (2) backs; (3) fronts; (4) roof; and (5) walls. For parts that consist of pairs of structures, the lower score obtained is used.

Ends
0 No end built
1 No doors on end
3 Door handle facing in wrong direction; doors set in too far
4 Door placed correctly on both ends

Backs
0 No backs
1 One of both windows omitted or wrong window used
2 Window placed on wrong level and/or not in middle half
3 Window on correct level and within middle half but not perfect; not set in correctly
4 Window placed correctly both vertically and horizontally on both backs

Fronts

0 No fronts
1 One or both windows omitted or wrong window used
2 Windows placed on wrong level and/or not in correct corner
3 Window on correct level and in correct corner, but not set back
4 Window placed correctly on both fronts

Roof

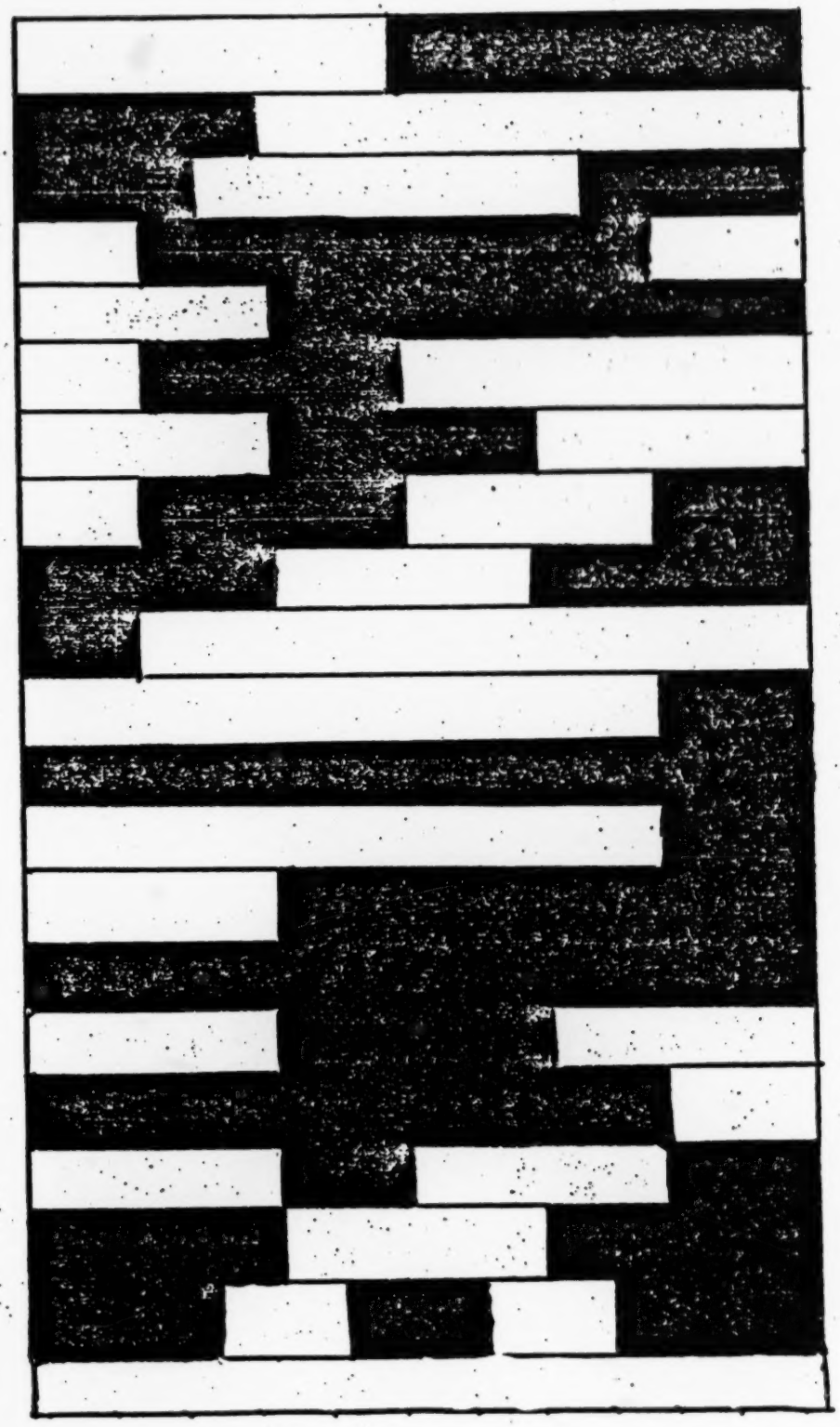
0 No roof
1 Wrong number of levels; holes in roof
2 Correct number of levels, but no terracing at first level (i.e., on top level of yellow bricks)
3 Roof correctly terraced but incorrectly interlocked
4 Roof constructed without errors

Walls -- Structure in general
0 No wall construction at all
1 Wrong number of levels OR wrong colors used (i.e., not red and yellow)
2 Correct number of levels but wrong color order
3 House built as two seperate but otherwise correct units; bases not joined
4 Perfect construction of all walls -- correct number of levels AND color order

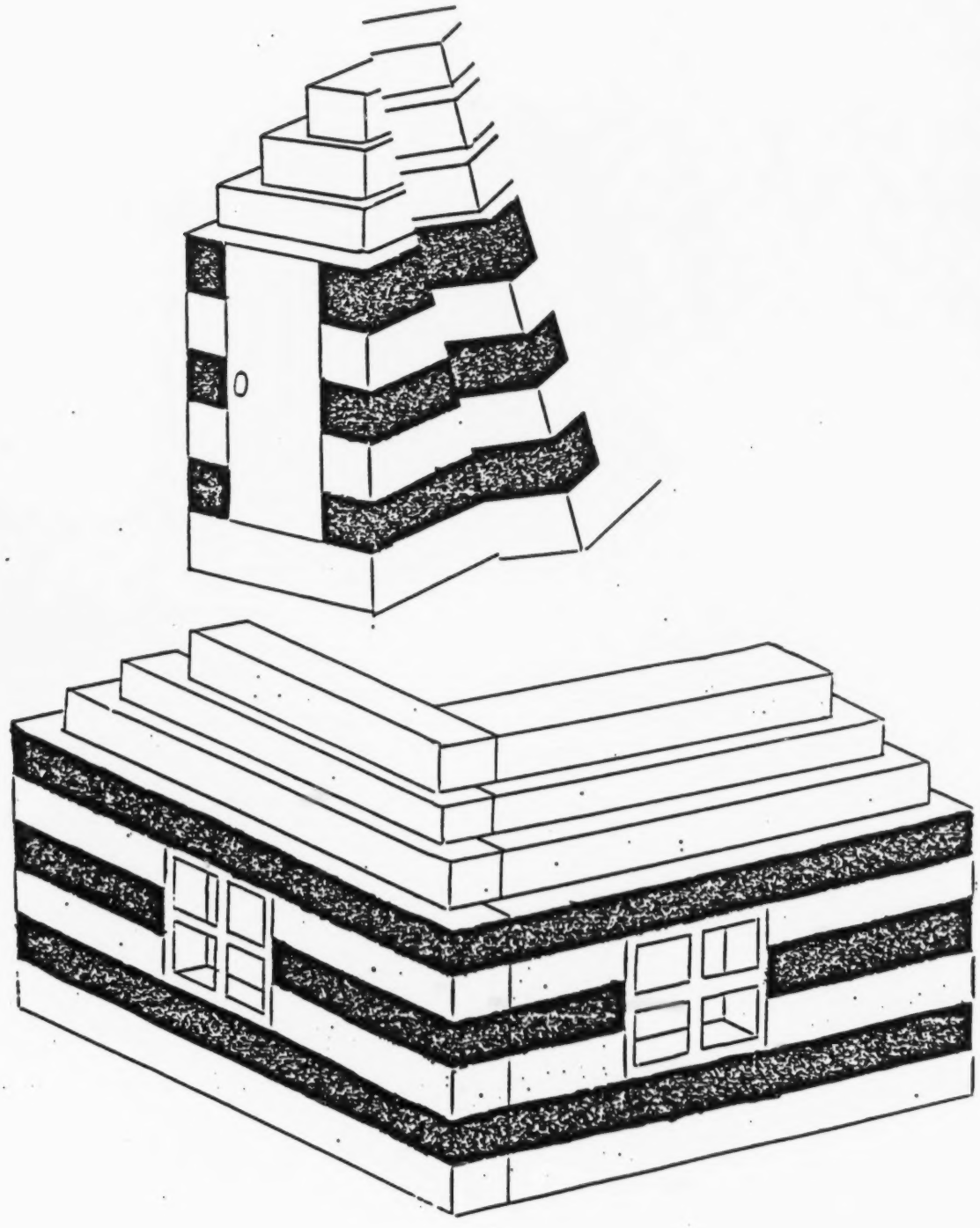
Scoring for Wall:

Each of 20 rows is determined to be either correct or incorrect.
Score is the number of correct rows constructed.

Model Building Task 1 -- Wall



Model Building Task 2 -- House



Spatial Perspectives Task 1 -- Picture Choice

Sources: Derived from Piaget and Inhelder (1956).

Materials:

Wooden support with dolls attached
Brio buildings, roofs, trees
Set of plain blocks
Description of two models
Photographs of models, 2 sets A-R and AA-RR
Computer, with "PIC.BAS" loaded

Items to be used:

Two models are to be constructed by the observers. The first is a village scene and the second is a much more abstract, urban scene. A drawing of the Brio village scene is attached. The "urban" scene was composed of plain blocks to represent a city block. For each, the pairs are to decide how the scene must look to 7 or 8 "dolls" which are placed around the models. The children choose the correct perspective from about 15 photos of the models, 8 of which are representational (i.e., are photographs of real model) and the remainder imaginary (photographs taken after some blocks were exchanged). The investigators can be contacted if actual photographs are needed.

Instructions:

The board with the model should be placed on a table and pushed up against the wall, so that the kids can't walk around it. Place it so that the RED DOLL is on your right. Say to the children:

"In these problems, we want to see if you can figure out how another person looks at the same scene you're looking at. There are two scenes for you to work with. The first one is this village in front of you. When you're finished with this one, we'll set up the second one for you. You must stay on this side of the model -- you cannot move around it.

"Here's what we want you to do. Look at the first item on the answer sheet and on the computer screen. It says RED. I want you to look through this pile of pictures together and try to decide which view the RED DOLL sees. Can you try this?"

(Let them work on this problem. Encourage them to work together, as usual. If they get it right, ask them if they think they understand what to do and skip the rest of this paragraph. If wrong, tell them they are wrong and to try again. If they still have no success, point to one feature that they can easily see and ask them to find a picture that shows that object the way the RED DOLL sees it. If it is still problematic, give up.)

"Type in the code of the picture from the back next to the word RED. The computer will let you know if you are right or wrong. If you're right, the computer will show the color of another doll. If you're wrong, try again. In each case, try to figure out which view the doll sees and enter your answer into the computer. There are 8 more dolls around the model. I'd like you to try and figure out what each one sees and type in the letter from the back of the picture. If there is no picture that is right, write NONE. For all of them, I want you to help each other in making a choice. When you have finished, you can start on the next model. Any questions?"

Scoring:

Number correct.

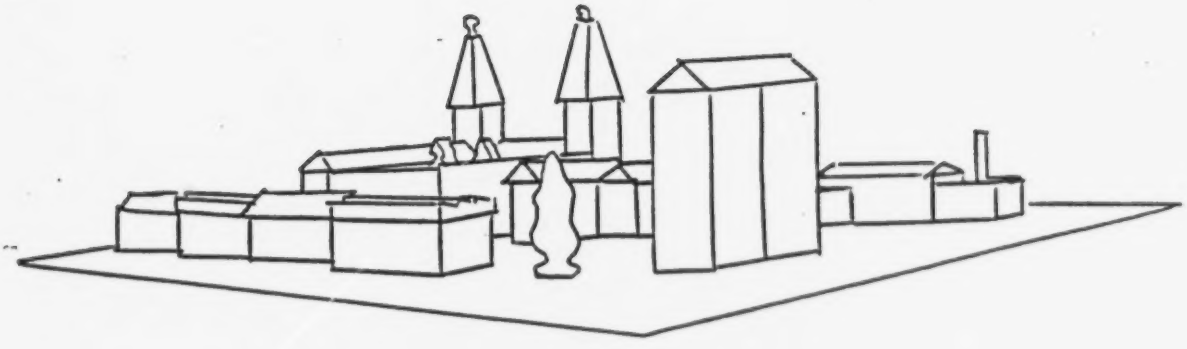
The correct answers are:

DOLLS	CITY	HOUSES
Red	GG	I
Purple	EE	E
Yellow	KK	A
Green	OO	K
Pink	AA	M
Brown	II	C
Black	MM	G
White	CC	P

Spatial Perspectives Task 1

Picture Choice

Drawing of Village Scene



Spatial Perspectives Task 2

Sources: Derived from Mugny and Doise (1978)

Materials (two sets of each):

Wooden support with lake
Brio buildings, roofs, trees
Transparent graph paper
Photographs of models

Items to be used:

Two practice items and ten "regular" items are shown in the booklet of photographs. The regular items are numbered in order of complexity: one practice and up to 10 regular items are used. A drawing of one setup is attached. The actual photographs are available from the investigators.

Instructions:

Children are placed facing one of the bases. The other base is placed on a table to their left at an angle of 90 degrees. Say to the children:

"These problems have two parts. In part 1, you need to set up a model by using the picture and layout as a guide. For example, (using the first practice item) we see two houses in the picture, one with a red roof and one with a black roof. Let's find those blocks in the box. Then we place the houses on the outlines on the board. Can you try that? (Help children do item until it is correct.) In part 2, you need to copy the model onto this other board, which is facing a different way. Pretend that you (one of the kids) are swimming in the lake. When you get out, you see these houses. Now, I want you to set up houses together on the other board so that if (other child) got out of that lake, he/she would see the houses in the same way. Can you try this one?" (Let kids do it and make sure they agree with each other about accuracy. Then go on to "real" items.)

"I have several problems of this kind for you to work together on. When you are ready, you can begin the first one. It is important that you work together on the problems until you both think you have finished it correctly Then you may get the next one. On each new problem, I will turn the board for your copy in a different direction."

"Some of the problems are harder than others. I want you to do them in order. It doesn't matter how many you do -- you'll probably only have time for one or two. It does matter that you do a good job and work together. And don't forget that you can't go to the next problem until you agree that the one you are working on is finished. Any questions?"

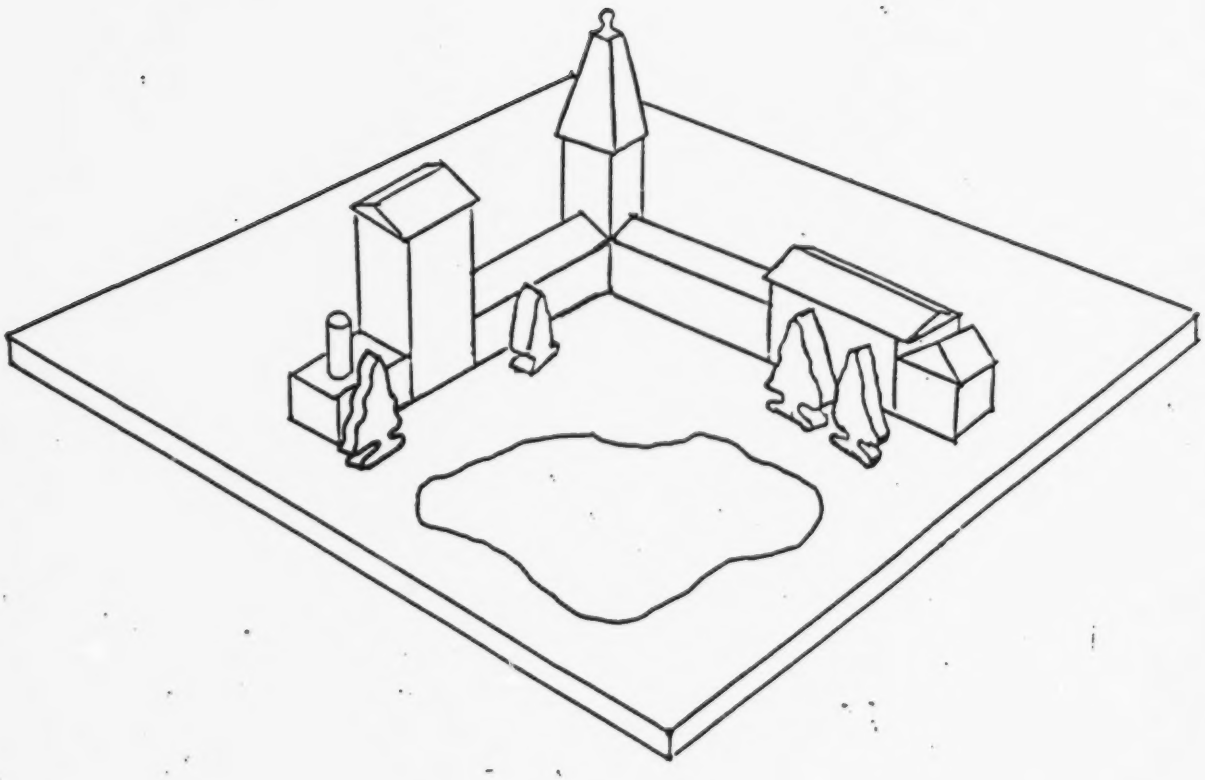
Complexity of task is varied by changing the orientation of the copy base and by changing the number of buildings and/or trees. In simple items, the orientation of the copy base is the same as the model base and the number of buildings/trees is small. In more complicated items, the base is rotated and the number of buildings/trees is increased.

Scoring:

We will use a variant of the scoring used by Bearison (1984). For each building and tree, determine whether or not it is facing the right edge of the paper (1 point) and whether it is in the correct quadrant of the paper (1 point); add the points for each object. Thus, each building/tree can have 0, 1, or 2 points. Add the points credited for each building for the total score per task. A template for each task is used for comparing the drawings.

Spatial Perspectives Task 2

Drawing from photograph of item 10



Spatial Perspectives Tasks 3 and 4 -- Map drawing

Sources: Snyder and Feldman (1977), Levin (1978), Snyder, Feldman, & LaRossa (1975)

Materials:

Model of rural scene for Task 3 (overhead view of map is attached)
Model of suburban scene for Task 4 (overhead view of map is attached)
Template for scoring (same as overhead views, which are attached)
Paper
Pencils

Items to be used:

The children draw a single map of the model.

Instructions:

The model should be placed on a low table, "thigh-level" for the students, and in front of the testing area. Desks or tables should be arranged outward from the near edge of the model or in a semicircle around the model so that all students can view model while seated. Students are permitted to walk up and examine the model, but should not stand over the model while drawing.

Each student should receive a standard lead pencil with an eraser and a sheet of 12" X 12" paper. Crayons, magic markers, colored pencils, pens, etc. should not be used.

Say to the children: "We are trying to find out the different ways that students draw a map. You should draw a map of this model, not some other place, and you should only use the paper and pencils we gave you. You can get up and look at the model whenever you want to, but please draw on this table."

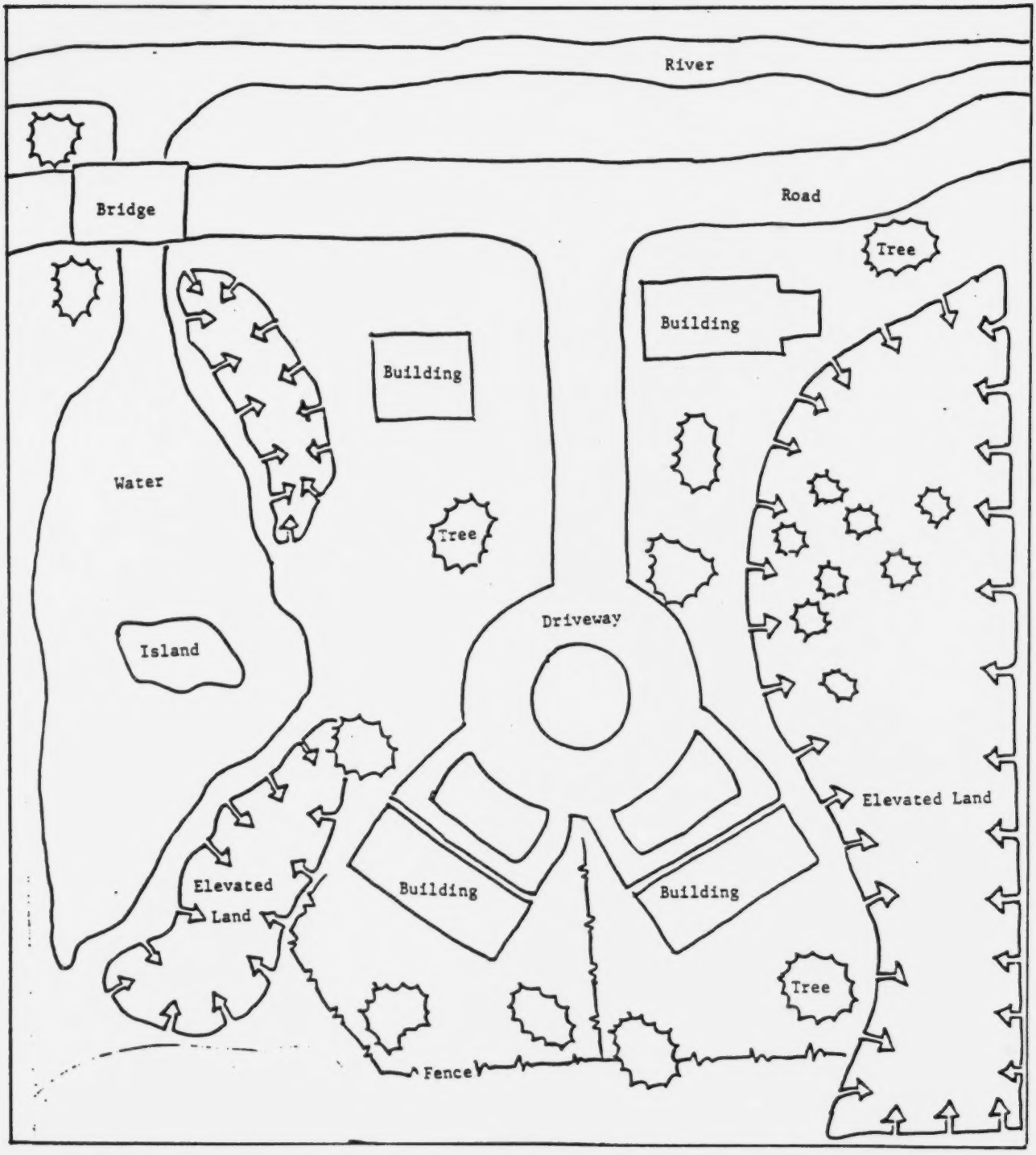
"Please try to remember two things: Work together and when you talk to each other, talk loud enough so that your voices will be recorded -- don't whisper!

Scoring:

Scoring is described in great detail in Snyder, Feldman, & LaRossa (1975). Briefly, 20 classifications are generated for each map drawing. From these 20 scores, several measures are derived, including model level, average level, level mixture, and bias.

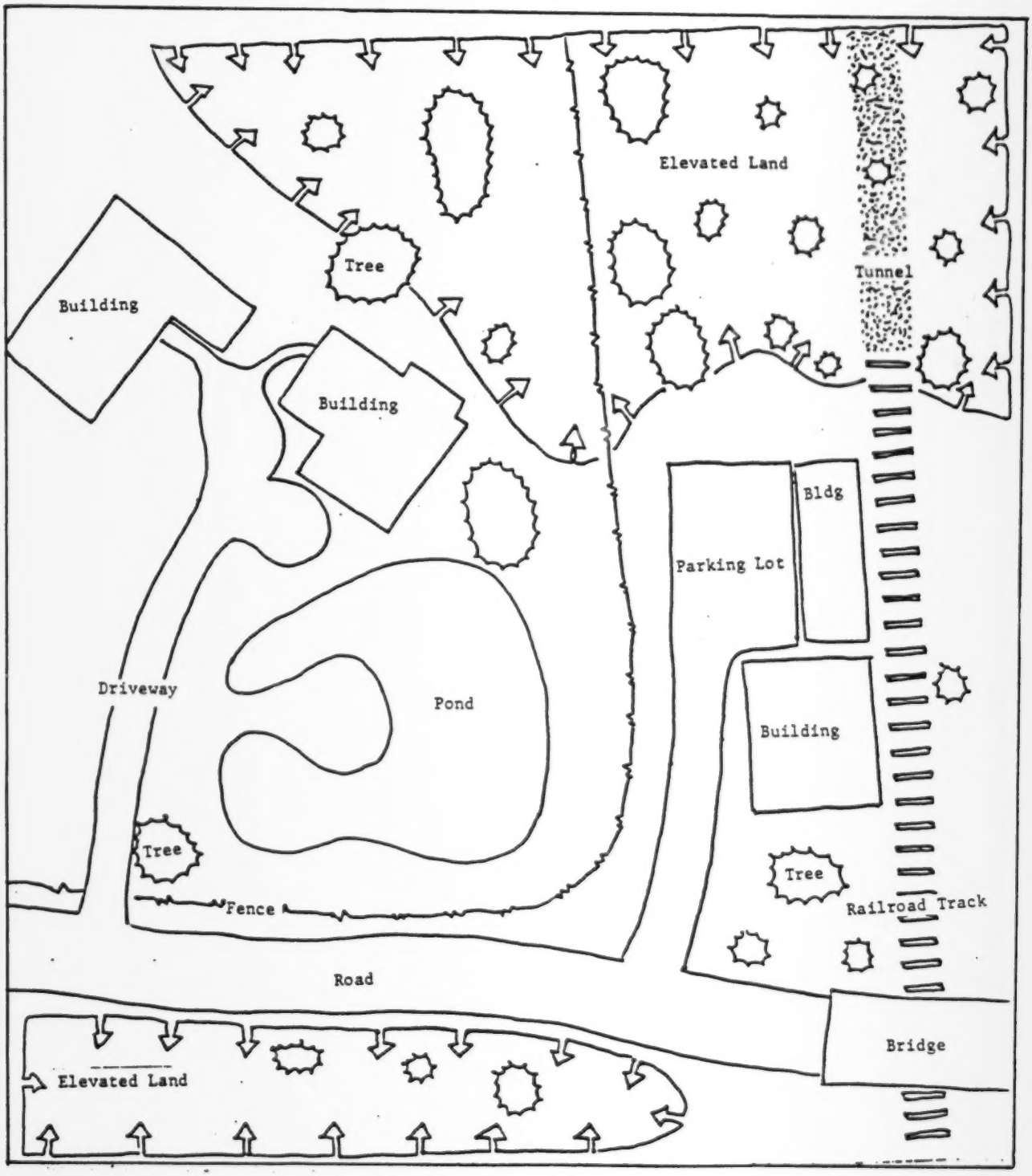
Spatial Perspectives Task 3

Map-drawing -- Suburban scene



Spatial Perspectives Task 4

Map-drawing -- Rural scene



Balance Scale Problem (6 sessions)

Sources: Siegler (1975), Klahr and Siegler (1978)

Materials:

Balance Scale
Blocks or paper cups
Weights (washers)
Binders with problems inserted
Answer sheets and pencils

Items to be used:

120 balance problems, attached, were developed and ordered such that the more difficult kinds of problems became more and more frequent as one progresses through the series. Before each session, the first ten of the twenty problems to be worked on by the next pair are inserted into the binder, followed by a sheet reading "STOP." The experimenter then inserts the next ten problems. For session 1, all pairs worked on problems 1 through 20. For the remaining sessions, each pair began with the problems they had missed during the session before and additional problems were added from the 120 problem sequence so that they had a total of 20 problems in each session. Thus, after session 1, pairs worked on different problems, depending on how many they had missed in the prior sessions.

Instructions:

Children are seated at a table with the balance scale, blocks, and two piles of weights (one pile for each child), within easy reaching distance. The black side of the scale should be on the children's left, to correspond to the drawings for each problem. The children are given one answer sheet to share.

Say to the children: "Today you are going to work together on some balance problems. There are 10 problems in this booklet. Each problem is presented as a picture of the scale with weights on it. Be sure the blocks are under your scale to keep it from moving; then set up your scale to look exactly like the scale in the picture. Work together to decide which side you think will go down, black or white, or whether you think it will stay balanced and neither side will go down when you remove the blocks. Write your answer on the answer sheet. Write "B" if you think the black side will go down, "W" if you think the white side will go down, or "=" if you think it will stay balance. Then remove the blocks. If you were right, circle your answer on the answer sheet. If your answer was wrong, just leave it. Then go on to the next problem When you finish the problems, stop and we'll come in to tell you what to do next. It doesn't matter how many problems you do, but how well you can agree on the answers. Do you have any questions? Be sure to work together and speak loudly so we can record your conversation?"

After the children complete the first ten problems the researcher returns and says to the children: "How did you do?" Listen to the answers. "Now I want you to spend some time trying to figure out what makes the balance scale work. You can experiment by putting different combinations of weights on the balance. You can go back to the problems you missed and try them again -- but don't change your answers on the answer sheet -- and you can make up your own problems. I'll come back in five minutes and give you ten more problems to do."

After allowing the children 5 minutes to experiment, the researcher returns and says, "What do you think makes the balance work?" Pursue this question, asking for examples or clarifications if necessary , until the children have made a reasonable attempt to answer clearly. While talking with the children, take the ten problems already done out of the folder and replace them with the next ten. When the children are done explaining the balance, return the binder to them and say, "Here are the last ten problems You do them the same way you did the first ten. Don't worry if you don't finish them all -- it's more important how hard you work. Any questions?"

Scoring:

Number correct.

Balance Scale Task -- Items

Type of Problem	L (left) Weight	R (right) Distance (from fulcrum)
BALANCE	L = R	L = R
WEIGHT (side with greater distance goes down)	L = R	L = R
DISTANCE (side with greater distance goes down)	L = R	L = R
CONFLICT-WEIGHT (side with greater weight goes down)	L > R	R > L
OR	R > L	L > R
CONFLICT-DISTANCE (side with greater distance goes down)	L > R	R > L
OR	R > L	L = R
CONFLICT-BALANCE (scale remains balanced)	L=R	L=R

	Problem Type[1]	Problem[2]		Answer[2]
1.	B	B3 = 1	W3 = 1	=
2.	W	B1 = 2	W1 = 3	W
3.	D	B1 = 1	W3 = 1	W
4.	CW	B2 = 1	W1 = 3	W
5.	CD	B2 = 3	W1 = 4	B
6.	D	B1 = 1	W4 = 1	W
7.	B	B1 = 2	W1 = 2	=

[1]Key to problem type:
B = balance
W = weight
D = distance
CW = conflict weight
CD = conflict distance
CB = conflict balance

[2]B = black, W = white

Balance Scale Task -- Items

Problem	Type	Problem		Answer
8.	CW	B3 = 2	W4 = 1	B
9.	CD	B2 = 5	W3 = 4	W
10.	W	B3 = 4	W3 = 2	B
11.	D	B3 = 2	W2 = 2	B
12.	CD	B1 = 4	W2 = 3	W
13.	W	B1 = 2 B2 = 1	W1 = 1 W2 = 1	B
14.	CW	B4 = 1	W3 = 4	W
15.	B	B1 = 2 B2 = 1	W1 = 2 W2 = 1	=
16.	D	B2 = 3	W3 = 3	W
17.	W	B1 = 2 B3 = 2	W1 = 2 W3 = 3	W
18.	CD	B3 = 4	W5 = 2	B
19.	B	B1 = 2 B3 = 2	W1 = 2 W3 = 2	=
20.	CW	B4 = 3	W5 = 2	B
21.	D	B4 = 3	W5 = 3	W
22.	W	B1 = 3 B4 = 2	W1 = 5 W4 = 2	W
23.	CD	B5 = 1	W1 = 3	B
24.	B	B2 = 3 B3 = 2	W2 = 3 W3 = 2	=
25.	CW	B2 = 2 B3 = 2	W4 = 2	B

Balance Scale Task -- Items

Problem	Type	Problem		Answer
26.	W	B2 = 4 B5 = 1	W2 = 5 W5 = 2	W
27.	CD	B4 = 2	W1 = 5	B
28.	CW	B4 = 3	W2 = 4 W3 = 5	W
29.	CD	B3 = 4	W1 = 2 W2 = 3	B
30.	D.	B4 = 4	W1 = 4	B
31.	CD	B1 = 2 B2 = 2	W3 = 5	W
32.	CW	B3 = 6	W4 = 2 W5 = 1	B
33.	D	B2 = 5	W1 = 5	B
34.	CD	B3 = 4 B4 = 1	W5 = 4	W
35.	B	B2 = 3 B5 = 4	W2 = 3 W5 = 4	=
36.	CD	B4 = 3	W1 = 2 W2 = 3	B
37.	W	B2 = 2 B3 = 4	W2 = 4 W3 = 6	W
38.	CW	B2 = 3 B3 = 6	W5 = 3	B
39	CD	B2 = 2 B3 = 3	W4 = 4	W
40.	D	B5 = 6	W2 = 6	B
41.	D	B3 = 2 B4 = 2	W1 = 2 W2 = 2	B

Balance Scale Task -- Items

Problem	Type	Problem		Answer
42.	W	B3 = 3	W3 = 3	
		B5 = 5	W5 = 3	B
43.	CD	B5 = 4	W1 = 4	
			W2 = 3	B
44.	B	B1 = 1	W1 = 1	
		B2 = 2	W2 = 2	
		B3 = 3	W3 = 3	=
45.	CW	B5 = 6	W3 = 6	
			W4 = 5	=
46.	CD	B2 = 3	W5 = 4	
		B3 = 3		W
47.	D	B4 = 3	W1 = 3	
		B5 = 3	W2 = 3	B
48.	CD	B2 = 4	W4 = 5	
		B3 = 3		W
49.	CD	B5 = 3	W1 = 4	
			W3 = 3	B
50.	CW	B3 = 1	W1 = 2	
		B4 = 1	W2 = 3	W
51.	CD	B5 = 5	W1 = 3	
			W4 = 5	B
52	D	B1 = 4	W4 = 4	
		B2 = 4	W5 = 4	W
53.	CD	B1 = 2	W3 = 2	
		B2 = 2	W4 = 1	W
54.	CD	B4 = 1	W2 = 3	
		B5 = 2	W3 = 2	B
55.	W	B2 = 1	W2 = 2	
		B4 = 3	W4 = 5	W

Balance Scale Task -- Items

	Problem Type	Problem		Answer
56.	CD	B1 = 3	W3 = 1	
		B2 = 3	W4 = 2	W
57.	B	B1 = 3	W1 = 3	
		B3 = 1	W3 = 1	
		B5 = 5	W5 = 5	=
58.	CB	B4 = 3	W3 = 4	=
59.	CD	B3 = 2	W1 = 5	
		B4 = 3	W2 = 4	B
60.	CW	B1 = 3	W3 = 1	
		B2 = 6	W4 = 2	B
61.	CD	B1 = 5	W3 = 4	
		B2 = 4	W4 = 3	W
62.	CW	B3 = 3	W4 = 2	
		B4 = 4	W5 = 2	B
63.	CD	B4 = 1	W1 = 2	
		B5 = 2	W2 = 2	B
64.	W	B1 = 5	W1 = 4	
		B5 = 6	W5 = 5	B
65.	CD	B1 = 5	W4 = 4	
		B2 = 3	W5 = 1	W
66.	B	B2 = 2	W2 = 2	
		B4 = 4	W4 = 4	
		B5 = 2	W5 = 2	=
67.	CD	B1 = 2	W3 = 1	
		B2 = 2	W5 = 1	W
68.	D	B3 = 5	W1 = 5	
		B4 = 5	W2 = 5	B
69.	CD	B3 = 3	W1 = 5	
		B5 = 5	W2 = 4	B
70.	CB	B3 = 5	W5 = 3	=

Balance Scale Task -- Items

	Problem Type	Problem		Answer
71.	CD	B3 = 2	W1 = 6	
		B5 = 5	W2 = 3	B
72.	W	B1 = 2	W1 = 3	
		B2 = 4	W2 = 5	W
73.	CD	B1 = 6	W3 = 5	
		B2 = 4	W4 = 3	W
74.	CB	B1 = 2	W2 = 1	=
75.	CW	B2 = 3	W4 = 3	
		B3 = 5	W5 = 1	B
76.	CD	B4 = 3	W2 = 6	
		B5 = 3	W3 = 3	B
77.	CD	B1 = 3	W4 = 2	
		B2 = 5	W5 = 3	W
78.	B	B3 = 5	W3 = 5	
		B4 = 4	W4 = 4	
		B5 = 3	W5 = 3	=
79.	CB	B2 = 3	W1 = 6	=
80.	D	B4 = 6	W2 = 6	
		B5 = 6	W3 = 6	B
81.	W	B3 = 4	W3 = 3	
		B4 = 4	W4 = 3	B
82.	CD	B1 = 5	W3 = 3	
		B2 = 3	W5 = 1	W
83.	D	B4 = 4	W2 = 4	
		B5 = 3	W3 = 3	B
84.	CD	B3 = 1	W1 = 4	
		B4 = 4	W2 = 4	B
85.	CB	B2 = 6	W3 = 4	=
86.	CW	B1 = 3	W4 = 1	
		B2 = 4	W5 = 1	B

Balance Scale Task -- Items

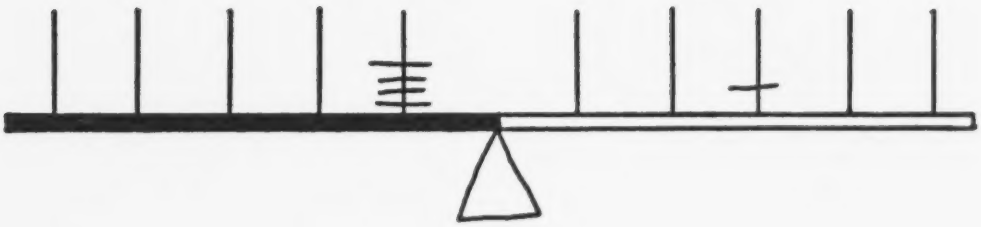
Problem	Type	Problem		Answer
87.	CB	B3 = 1	W1 = 3	=
88.	CD	B1 = 5	W4 = 4	
		B2 = 6	W5 = 4	W
89.	CD	B2 - 6	W4 = 3	
		B3 = 5	W5 = 4	W
90.	CB	B2 = 1	W1 = 2	
		B4 = 3	W3 = 4	=
91.	CD	B3 = 4	W1 = 3	
		B4 = 3	W2 = 6	B
92.	CB	B1 = 5	W3 = 2	
		B2 = 3	W5 = 1	=
93.	CW	B2 = 5	W4 = 3	
		B3 = 5	W5 = 2	B
94.	CD	B1 = 6	W4 = 3	
		B2 = 6	W5 = 3	W
95.	CB	B4 = 1	W1 = 4	=
96.	CB	B4 = 3	W2 = 5	
		B5 = 2	W3 = 4	=
97.	D	B2 = 3	W4 = 2	
		B3 = 1	W5 = 2	W
98.	CD	B4 = 4	W1 = 3	
		B5 = 3	W3 = 6	B
99.	CB	B1 = 5	W5 = 1	=
100.	W	B2 = 6	W2 = 4	
		B3 = 4	W3 = 4	B
101.	CD	B2 = 6	w4 = 2	
		B3 = 4	W5 = 4	W
102.	CB	B1 = 4	W4 = 1	=
103.	CB	B1 = 6	W3 = 2	
		B2 = 4	W4 = 2	=

Balance Scale Task -- Items

Problem Type		Problem		Answer
104.	D	B2 = 2 B3 = 6	W4 = 6 W5 = 2	W
105.	CD	B4 = 1 B5 = 3	W1 = 5 W2 = 4	B
106.	CB	B2 = 6 B3 = 5	W4 = 3 W5 = 3	=
107.	CW	B1 = 6 B2 = 6	W4 = 1 W5 = 2	B
108.	CB	B4 = 1 B5 = 1	W3 = 3	=
109.	CD	B1 = 2 B2 = 5	W3 = 3 W5 = 2	W
110.	CB	B1 = 2	W2 = 1	=
111.	CB	B3 = 3	W1 = 3 W2 = 3	=
112.	CD	B3 = 6 B4 = 2	W1 - 6 W2 = 4	B
113.	CD	B2 = 5 B3 = 6	W4 = 3 W5 = 4	W
114.	CB	B1 - 3	W3 = 1	=
115.	CW	B4 = 2 B5 = 1	W1 = 4 W3 = 5	W
116.	CB	B5 = 1	W1 = 5	=
117.	CB	B3 = 1	W1 = 1 W2 = 1	=
118.	CD	B3 = 6 B5 = 3	W1 = 5 W2 = 6	B
119.	CB	B2 = 1 B3 = 1	W5 = 1	=
120.	D	B3 = 6	W1 = 3 W2 = 5	

Balance Scale Tasks -- Sample Set-up Sheet

S E T U P T H E B A L A N C E L I K E T H I S :



W H I C H S I D E W I L L D R O P W H E N
Y O U R E M O V E T H E B L O C K S ?

Pretest/posttest Administration

Math pretests

Give the children the test booklets and a pencil. Tell them to work through as many problems as they can and that they can work for 30 minutes. Remind them that some are addition problems and some are multiplication problems, so they should read the problems carefully. The test booklet is attached.

Spatial pretests

The materials for these tasks are available from the investigators.

There are 4 groups of tasks. The map drawing task will take 20 minutes and the three remaining tasks will take 5 minutes each. So, again, divide the spatial group into halves and have the first half work on the map task while the others rotate through the other three tasks. Then have them switch. Give them an answer booklet and have them keep it with them as they do all of the tasks. Beforehand, set up the tasks at 4 separate tables:

1. Picture choice task -- One item with plain blocks. Use photos and green paper with layout to help set up. Put the corner that the RED DOLL is on toward the subject, on his or her left. Place dolls around structures as indicated on green paper. Put photos to be used in making choices next to blocks. These should be set up so that kids cannot walk around them. Score is number correct.

Say to the children: "In this problem, we want to see if you can figure out how another person looks at the same scene you're looking at. There are two scenes for you to work with. When you're finished with this one, you can do the next one. You must stay on this side of the model -- you cannot move around it.

"Here's what we want you to do. Look at the first item on the answer sheet. It says RED. I want you to look through this pile of pictures and try to decide which view the RED DOLL sees. Can you try this?

"Write down the letters on the back of the picture you think is right. If there is no picture that is right, write NONE. Any questions?"

2. Orientation task -- Set up the Brio blocks as indicated in the photographs and by the green paper layout. The back of the layout indicates which side should be closest to the subjects. Make sure you orient them toward the kids as indicated on the bottom of the paper. Help them find the right page in their test booklet and tell them not to turn either the model or their pages around.

Tell the child: "I want you to use the blocks from this pile and copy the model onto your paper, which has the lake in a different place. Pretend that you are swimming in the lake. When you get out, you see these houses §point to model1. Now, I want you to set up houses on your paper so that if you got out of that lake, you would see the houses in the same way. When you have them set up, I will trace the buildings on the paper for you. Then you can do the other model in the same way."

Score as described in Spatial Perspectives Task 2.

3. Construction task -- For this task, you need the drawings of an abstract building and a bag of Legos that corresponds with the drawings. Give the child the bag and the drawing and say, "For this project, you should build a building out of Legos. Here is a drawing of the building. Use the Legos in front of you to build it so that it looks just like the one in the picture. When you are done, write your name on the label and stick it on to your model."

Score as described in Model Copy Tasks.

4. Map drawing task -- For this task, you need the Suburban model, pencil, and the page in the test booklet for the map.

The model should be placed on a low table, "thigh-level" for the students, and in front of the testing area. Desks or tables should be arranged outward from the near edge of the model or in a semicircle around the model so that all students can view the model while seated. Students are permitted to walk up and examine the model, but should not stand over the model while drawing.

Say to the children: "We are trying to find out the different ways that students draw a map. You should draw a map of this model, not some other place, and you should only use the paper and pencils we gave you. You can get up and look at the model whenever you want to, but please draw on this table. You will have only 20 minutes to do this."

Score as in Spatial Perspectives Tasks 3 and 4.

Balance Scale Test

The test booklet of 30 items for the Balance Scale Test is attached. The experimenter passes out the booklet and explains that the children should circle the answer that indicates what the balance will do when the indicated number of weights are placed on the pegs. The experimenter stands in front of the children and demonstrates each problem with the actual balance scale and weights, without actually letting the scale arms move. Score is number correct.

Name KEY Date _____

Teacher _____ Observers _____

NUMBER PROBLEMS, Pretest

6 + 2 = 8 3 + 4 = 7

1 x 1 = 1 2 x 3 = 6

4 x 5 = 20

1. The post office hired 10 new mail sorters. Three of them were women. How many were men?

7

2. There are 48 sports cars and station wagons in a used car lot. If there are 36 sports cars, how many station wagons are there?

12

3. Kay plays 3 games of tennis each week. How many games of tennis does she play in 4 weeks?

12

4. There are 5 shelves of books. Each shelf has 7 books. How many books are there altogether?

35

5. Jim bought 4 packages of balloons. Each package had 10 balloons. How many balloons did he have altogether?

40

6. Terry writes one label for every jar of jam made. How many labels does he write for --

   a) 3 jars? 3

   b) 5 jars? 5

   c) 10 jars? 10

7. Three dogs eat 3 cans of dog food. How many cans do --

   a) 6 dogs eat? 6

   b) 9 dogs eat? 9

   c) 11 dogs eat? 11

8. Marty's family drinks 2 gallons of milk a week. How many gallons do they drink in --

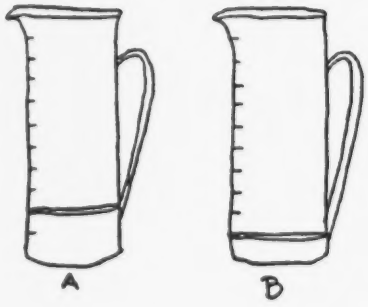
   a) 3 weeks? 6

   b) 5 weeks? 10

   c) 12 weeks? 24

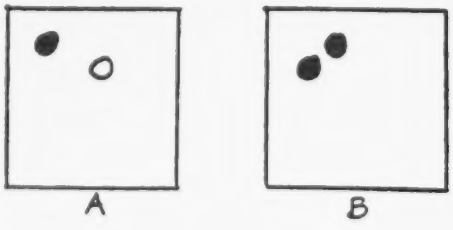
9. Pitcher A has 1 cup orange juice and 1 cup water. Pitcher B has 1 cup orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A?

   1



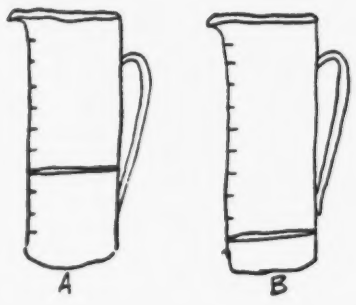
10. BOX A has 1 black chip and 1 white chip. BOX B has 2 black chips. How many white chips does BOX B need to make the mixture of black and white chips the same as BOX A?

    2



11. Pitcher A has 2 cups orange juice and 2 cups water. Pitcher B has 1 cup juice. How much water should be added to Pitcher B to make it taste like Pitcher A?

1



6 + 39 = 45

57 + 46 = 103

3 × 9 = 27

62 × 3 = 186

156 + 87 = 243

12. Together, Susan and Ryan have 83 comic books. If Ryan has 37 comic books, how many does Susan have?

46

13. Terry received 2 sets of stickers for her birthday which added 34 to her collection. The first set contained 23 stickers. If she now has 60 stickers, how many were in the second set?

11

14. Beth and Andy sold 82 boxes of candy last weekend. They sold 47 boxes on Saturday and 35 on Sunday. If Andy sold 23 boxes, how many did Beth sell in all?

59

2/26/85

Name_____Pretest, p. 4

15. Jessica jumps rope 5 minutes every day. How many minutes does she jump in 7 days?

35

16. Notebooks were on sale for $0.48 each. How much does Terry spend for 4 notebooks?

$1.92

17. At the Fireside Inn, Mark puts 2 red flowers and 1 yellow flower in a vase on each table. How many yellow flowers does he need if he has --

a) 4 red flowers? 2

b) 6 red flowers? 3

c) 10 red flowers? 5

d) 12 red flowers: 6

18. To make dark green paint, the paint store mixes 2 cans of yellow paint with 6 cans of blue paint. How many cans of blue paint should be mixed with --

a) 4 cans of yellow paint? 12

b) 6 cans of yellow paint? 18

c) 10 cans of yellow paint? 30

d) 1 can of yellow paint? 3

Name_____Pretest, p. 5

19. If 2 pencils cost 5 cents, how much money do you need for --

a) 4 pencils? 10¢

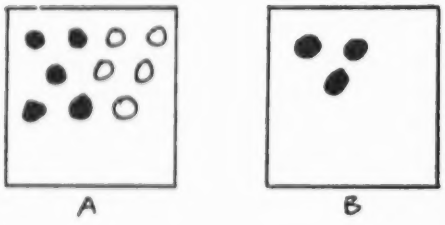
b) 6 pencils? 15¢

c) 8 pencils? 20¢

d) 12 pencils? 30¢

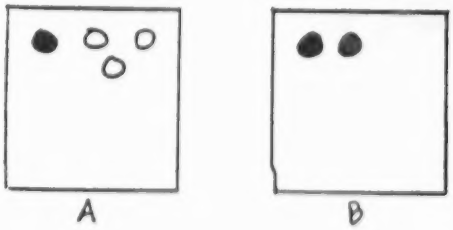
20. BOX A has 5 black chips and 5 white chips. BOX B has 3 black chips. How many white chips does BOX B need to make the mixture of black and white chips the same as BOX A?

3



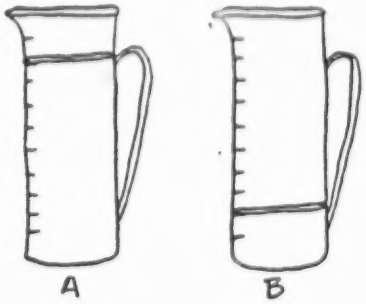
21. BOX A has 1 black chip and 3 white chips. BOX B has 2 black chips. How many white chips does BOX B need to make the mixture of black and white chips the same as BOX A?

6



22. Pitcher A has 3 cups orange juice and 6 cups water. Pitcher B has 2 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A?

4



Name_____Pretest, p. 6

397 + 327 = 724

41 x 73 = 2993

312 x 2 = 624

6291 + 2559 = 8850

113 x 23 = 2599

23. Jane bought 675 feet of blue and red ribbon. She used 350 feet to make party decorations. If 249 feet of the ribbon she bought were blue, how many feet were red?

426

24. A movie theater sold 8527 tickets last weekend. 6738 tickets were sold on Saturday. That is 4949 more than were sold on Sunday. Altogether, how many tickets were sold on Sunday?

1789

25. There are 46 rows in a movie theater with 9 seats in each row. If all the seats are filled, how many tickets were sold?

414

26. There are 144 eggs in a crate. If there are 8 crates, how many eggs are there in all?

1152

27. Skyline Drive Center contains 12 office buildings. There are 225 people working in each building. How many people work at Skyline Center?

2700

28. Jane makes salad dressing with 6 tablespoons of oil and 2 tablespoons of vinegar. How many tablespoons of vinegar does she need with --

a) 12 tablespoons oil? 4

b) 18 tablespoons oil? 6

c) 21 tablespoons oil? 7

d) 24 tablespoons oil? 8

29. The Nut House puts 3 lbs. of almonds and 5 lbs. of peanuts in its nut mixture. How many lbs. of peanuts will there be if there are --

a) 6 lbs. of almonds? 10

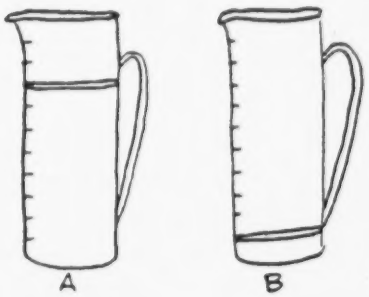
b) 9 lbs. of almonds? 15

c) 12 lbs. of almonds? 20

d) 18 lbs. of almonds? 30

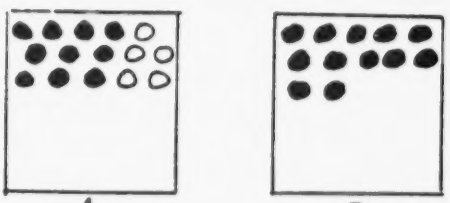
30. Pitcher A has 2 cups orange juice and 6 cups water. Pitcher B has 1 cup juice. How much water should be added to Pitcher B to make it taste like Pitcher A?

3



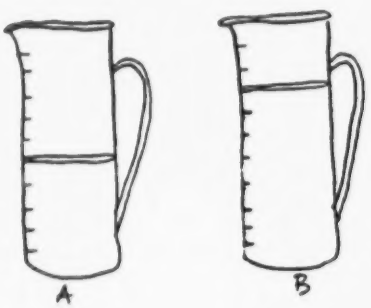
31. BOX A has 10 black chips and 5 white chips. BOX B has 12 black chips. How many white chips does BOX B need to make the mixture of black and white chips the same as BOX A?

6



32. Pitcher A has 4 cups orange juice and 1 cup water. Pitcher B has 8 cups orange juice. How much water should be added to Pitcher B to make it taste like Pitcher A?

2



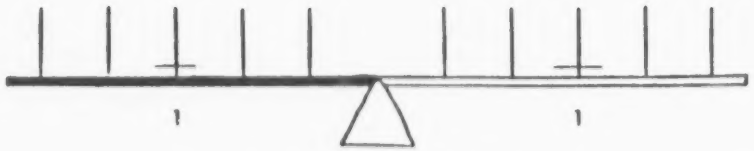
NAME _____

SCHOOL _____ TEACHER _____

BALANCE SCALE PROBLEMS

Pretest, Fall 1985

1.



A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

2.

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

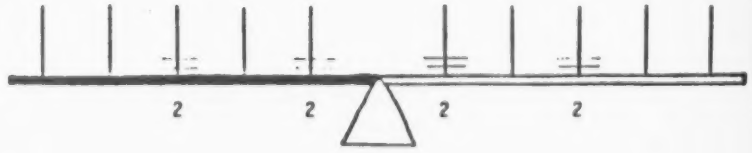
3.

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

4.

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

5.

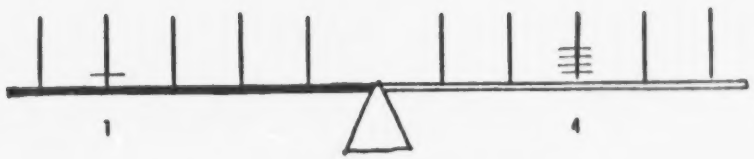


A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

6.

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

7.



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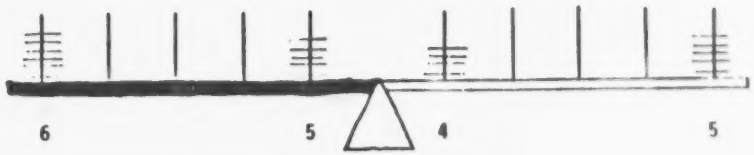
B. The WHITE side goes down.
C. The balance stays the SAME.

8.



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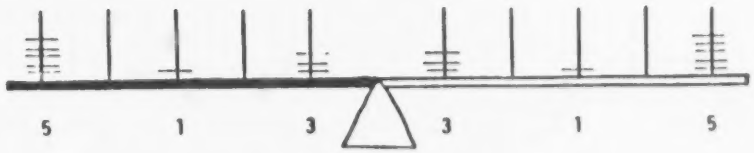
B. The WHITE side goes down.
C. The balance stays the SAME.

9.



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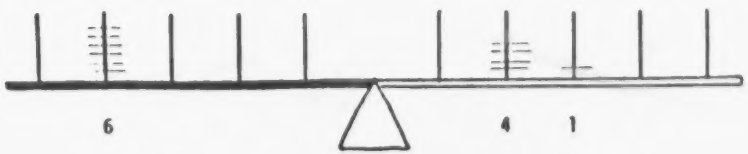
B. The WHITE side goes down.
C. The balance stays the SAME.

10.



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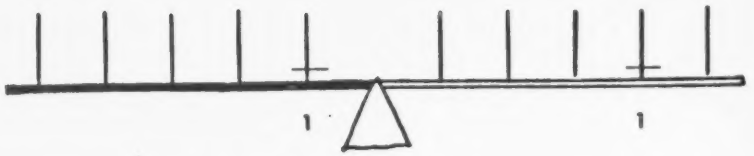
C. The balance stays the SAME.

11.



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C. The balance stays the SAME.

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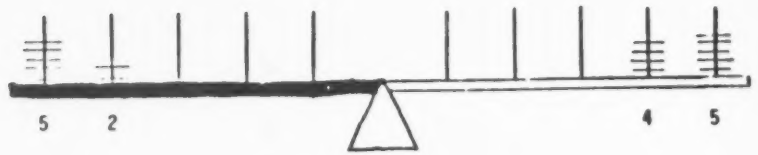

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C. The balance stays the SAME.

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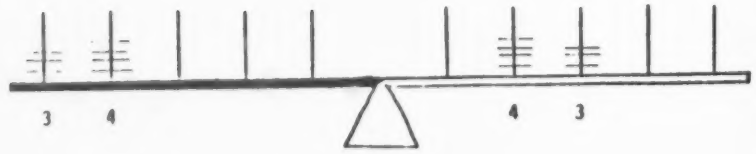

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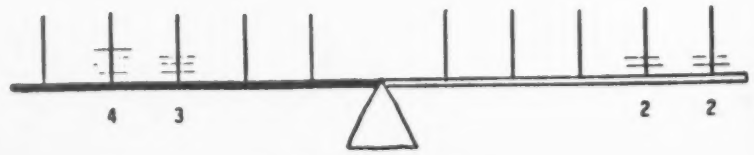
14.



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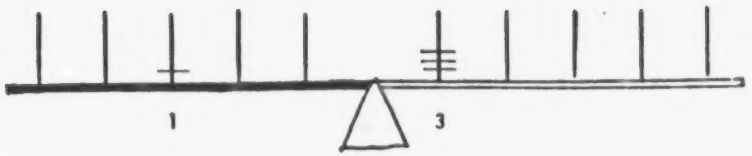
C. The balance stays the SAME.

15.



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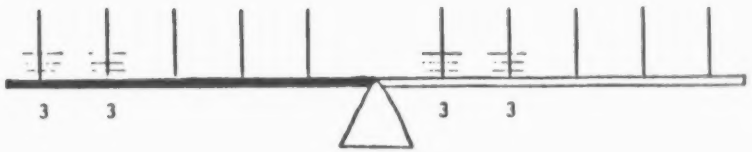
C. The balance stays the SAME.

16.

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

17.



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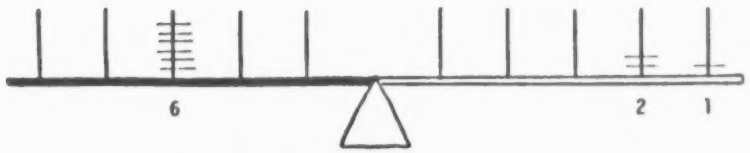
C. The balance stays the SAME.

18.



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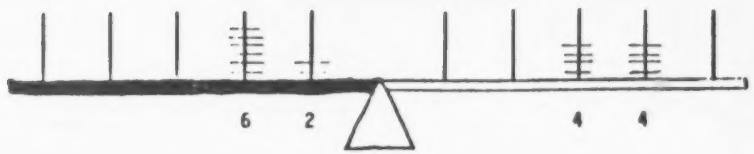
C. The balance stays the SAME.

19.

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

20.



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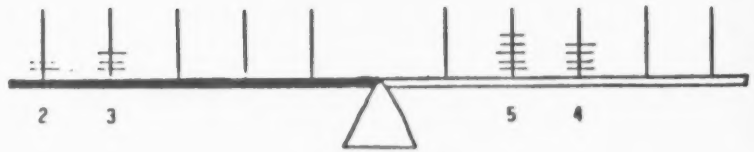
21.



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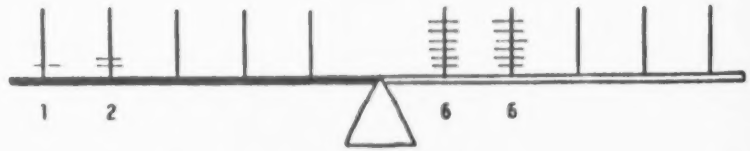

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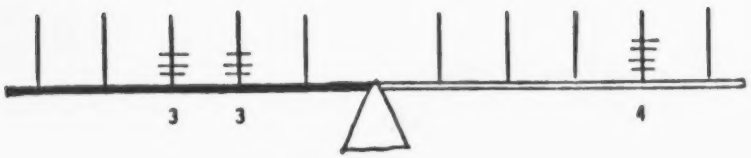
C. The balance stays the SAME.

23.



A. The BLACK side goes down.
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C. The balance stays the SAME.

24.

A. The BLACK side goes down.
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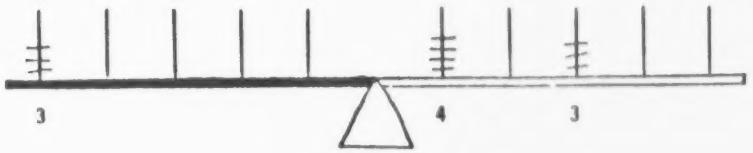
C. The balance stays the SAME.

25.



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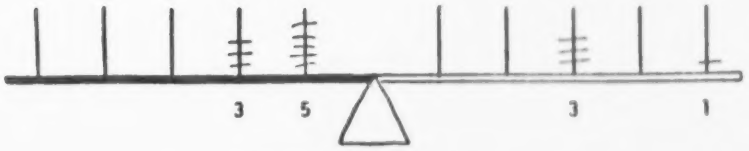
B. The WHITE side goes down.
C. The balance stays the SAME.

26.



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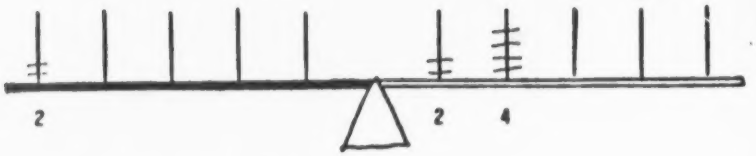

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28.

A. The BLACK side goes down.
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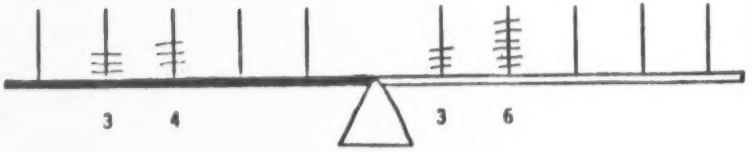
29.



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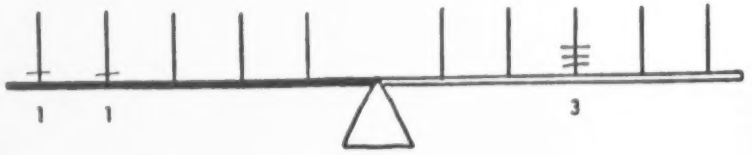

A. The BLACK side goes down.
B. The WHITE side goes down.
C. The balance stays the SAME.

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